International Journal of Research in Advent Technology, Vol.6, No.8, August 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

On Mathematical Analysis for LRS Bianchi Type II Cosmogical Models in Stiff and Anti-stiff Fluid in Modified Theory of General Relativity

R.K. Dubey[1] and Shishir Kumar Srivastava[2]

[1]*Department of Mathematics Govt. Model Science College, Rewa (M.P.) India.*
[2]*Department of Mathematics Ganpat Sahai P.G. College, Sultanpur (U.P.) India.*

Abstract- We consider a locally rotationally symmetric Bianchi type II string cosmological model in the presence of stiff and anti-stiff fluids are discussed. To get a deterministic model, it is assumed that$\sigma \propto \theta$, whereθ is the scalar of expansion, σ is the shear tensor. In the present study, we calculate two cases (1) and (2), where and are the rest energy density and the pressure of fluid. The physical and geometrical behaviors of the models are also discussed.
Keywords- Stiff fluid, Anti- stiff fluid, Bianchi type II models, cosmic strings.

MSC2010 CLASSIFICATIONS-83C05, 83C15

PACS Number: 98.80.Jk, 04.20,-q

1. INTRODUCTION

The recent observation shows that $\Lambda \sim 10^{-55} cm^{-2}$ $While$ the particle physics predication for Λ is greater than this value by a factor of order 10^{120}. The latest observation of large scale structure (LSS) and cosmic microwave back ground radiation (CMBR) confirmed that the universe is highly homogeneous and isotropic on large scales. In general it has been observed that anisotropic model do not become sufficiently isotropic as they evolve into future. Cosmology is the scientific study of large scale properties of the universe as a whole cosmology is study of the motion of crystalline objects the origin of the universe is greatest cosmological

Mystery even today, Letelier[1]and stachel[5] have initiated the general relativistic treatment of the strings. Bali and Anjali[2] have investigated Bianchi type I magnetized string cosmological model in general relativity

Bali and Dave[4] have investigated some Bianchi type IX string. Asseo and Sol[6] emphasized the importance of Bianchi type II. Later on number of authors has discussed cosmological solution with bulk viscosity in various contents as Bali and Upadhyay[7]·Zel' dovich[3], believed that cosmic string have given rise to density perturbations. Which leads to the formation of galaxies Pradhan and kumhar[8] studied LRS Bianchi type II bulk viscous universe with decaying vacuum energy density Lambda. Raoet al[10] studied Exact Bianchi type II, VIII, and IX string cosmological model in Saez Ballester theory gravitation.

To get a determine solution, they we assume that $\sigma \propto \theta$ where $\sigma$is shear and θ is scalar of expansion and which lead, to α_1=$\alpha_2^l$where $l$is constant. Recently, Tyagiet al[11], have investigated Bianchi type II string cosmological model assuming the condition$\rho = n\mu$ whereρ isthe energy density and μ is the string tension and n is constant. In this paper we have investigated Bianchi type II string cosmological models for stiff and ant stiff fluid distribution under two conditions

$(i)\ p - \rho = 0\ and$

$(ii) p + \rho.$

The physical and geometrical behaviors of the model are also discussed.

2. METRIC AND FIELD EQUATIONS

We consider the LRS Bianchi type II space time Metric in the form.

$ds^2 = -dt^2 + (\alpha_2 dx_1 + \alpha_2 x_3 dx_2)^2 + \alpha_1 (dx_2^2 + dx_3^2)$ (1)

Where α_1 and α_2 are functions of t only.

The energy momentum tensor $T_\alpha^\beta$for a cloud of string with perfect fluid distribution is taken as

$T_\alpha^\beta = (\rho + p) v_\alpha v^\beta + p g_\alpha^\beta - \mu x_\alpha x^\beta$

(2)

$\rho = \rho_p + \mu$

(3)

Where ρ is the energy density for cloud of string with particle attached to them. $\theta = v_{;l}^l$is the scalar of expansion, $\rho_p$is the particle energy density, μ is the string tension density $v^\alpha$is the four velocity vector of particle and x^α is the unit space like vector representing the direction of string satisfying.

$v_\alpha v^\alpha = x_\alpha x^\alpha = -1, v^\alpha x_\alpha = 0$

(4)

In a co moving system we get

$v^{\alpha} = (0,0,0,1), x^{\alpha} = \left(\frac{1}{\alpha_2}, 0,0,0\right)$ (5)

The expression for scalar of expansion θ and shear scalar σ are

$\theta = v^{l}_{;l} = \frac{\dot{\alpha}_1}{\alpha_1} + \frac{2\dot{\alpha}_2}{\alpha_2} = 3H$

(6)

$\sigma^2 = \frac{1}{2}\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{1}{3}\left(\frac{\dot{\alpha}_1^2}{\alpha_1^2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} - \frac{2\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2}\right)$ (7)

The spatial volume V and the average scale factor $\xi(t)$ is given by

$V = \xi^3(t) = \alpha_1\alpha_2^2$

(8)

$\xi(t) = (\alpha_1\alpha_2^2)^{\frac{1}{3}}$

(9)

The Hubble parameter defined as

$H = \frac{\dot{\xi}}{\xi} = \frac{1}{3}\left(\frac{\dot{\alpha}_1}{\alpha_1} + \frac{2\dot{\alpha}_2}{\alpha_2}\right)$

(10)

The Einstein's Field equation for a system of strings are given by Letelier[1]

$R^{\beta}_{\alpha} - \frac{1}{2} Rg^{\beta}_{\alpha} = -T^{\beta}_{\alpha}$

(11)

For the metric (1), Einstein's Field equation can be written as

$\frac{\ddot{\alpha}_1}{\alpha_1} + \frac{\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} + \frac{\alpha_2^2}{4\alpha_1^4} = \rho$

(12)

$\frac{2\ddot{\alpha}_1}{\alpha_1} + \frac{\dot{\alpha}_2}{\alpha_1^2} - \frac{3}{4}\frac{\alpha_2^2}{\alpha_1^4} = p - \mu$

(13)

$\frac{\dot{\alpha}_1}{\alpha_1^2} + \frac{2\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} - \frac{\alpha_2^2}{4\alpha_1^4} = -\rho$

(14)

Here an over dot (.) denote, ordinary differentiation with respect to time t.

The particle energy density is given by

$\rho_p = \frac{\ddot{\alpha}_1}{\alpha_1} - \frac{\ddot{\alpha}_2}{\alpha_2} - \frac{3\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} - \frac{\alpha_2^2}{4\alpha_1^4}$

(15)

3. SOLUTION OF FIELD EQUATIONS

The Field equations (13)-(14) are system of three equation with five unknown $\alpha_1, \alpha_2, \mu, \rho$ and p. Two additional constraints, relating these parameters are required to obtain explicit solutions of the system. In this paper Bali et. al^2, Pradhanet. al^3 and Amir chash et. al^4. We assume there σ is proportional to expansion θ, $\sigma \propto \theta$. leads to

$\alpha_1 = \alpha_2^{l}$

(16)

Relation between metric potentials $\alpha_1, \& \alpha_2$ here$l$ is a constant.

Adding equation (12) and (14) we get

$\frac{\ddot{\alpha}_1}{\alpha_1} + \frac{\ddot{\alpha}_2}{\alpha_2} + \frac{3\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} + \frac{\dot{\alpha}_1^2}{\alpha_1} = p - \rho$

(17)

Again using equation (12) and (14) we get

$\frac{\ddot{\alpha}_1}{\alpha_1} + \frac{\ddot{\alpha}_2}{\alpha_2} - \frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} - \frac{\dot{\alpha}_1^2}{\alpha_1^2} + \frac{\alpha_2^2}{2\alpha_1^4} = p + \rho$

(18)

Case-(1); For still fluid

In this case

$p - \rho = 0$ (19)

From equation (16) we take

$l = 1$

$$\alpha_1 = \alpha_2$$

From equation (16) and (17) we get

$\frac{2\ddot{\alpha}_2}{\alpha_2} + \frac{4\dot{\alpha}_2^2}{\alpha_2} = 0$

$\frac{\ddot{\alpha}_2}{\alpha_2} + 2\frac{\dot{\alpha}_2^2}{\alpha_2} = 0$

$\frac{\ddot{\alpha}_2}{\alpha_2} + 2 \times (1)\frac{\dot{\alpha}_2^2}{\alpha_2} = 0$

$\frac{\ddot{\alpha}_2}{\alpha_2} + 2l\frac{\dot{\alpha}_2^2}{\alpha_2} = 0$ (20)

Integrating equation (20) we get

$\frac{\alpha_2^{2l+1}}{2l+1} = k_1t + k_2$ (21)

$\alpha_2^{2l+1} = (2l+1)(k_1t + k_2)$

$\alpha_2 = [(2l+1)(k_1t + k_2)]^{\frac{1}{2l+1}}$ (22)

Where k_1 and k_2 are constant of integration.

From equation (16) and (22) we get

$\alpha_2 = [(2l+1)(k_1t + k_2)]^{\frac{1}{2l+1}}$

(23)

Hence using transformation

$\tau = k_1t + k_2, x_1 = X_1, x_2 = X_2, x_3 = X_3$Of coordinates metric (1) can be written as

$ds^2 = -\frac{d\tau^2}{k_1^2} + [(2l+1)\tau]^{\frac{2}{2l+1}}(\mathrm{d}X_1 + X_3\mathrm{d}X_2)^2 + [(2l+1)\tau]^{\frac{2l}{2l+1}}(dX_2^2 + d\mathrm{X}_3^2)$

(24)

4. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

The energy density ρ, the string tension density μ, the scalar expansion θ, the shear & scalar σ, ρ_p the particle energy density

And spatial volume V are respectively given by

$\rho = p\frac{1}{4}[2l+1)\tau]^{\frac{2(1-2l)}{2l+1}} - \frac{l(lk_1+2)}{[(2l+1)\tau]^2}$

(25)

$\mu = [(2l+1)\tau]^{\frac{2(1-2l)}{2l+1}} + \frac{[2l(k_2-1)+l^2 k_1(k_1+1]}{[(2l+1)\tau]^2}$
(26)
$\theta = \frac{1}{\tau}$
(27)
$\sigma = \frac{l-1}{\sqrt{3}}[\frac{1}{(2l+1)\tau}]$
(28)
$$\rho_p = \frac{l[2-k_1[l+4)]}{[(2l+1)\tau]^2} - \frac{1}{4}[(2l+1)\tau]^{\frac{2(1-2l)}{2l+1}}$$
$V = (2l+1)\tau$

Case (2); Anti- stiff fluid
In this case
$p + \rho = 0$ (29)
Using equation (16) and (18) and (29) we get.
$\frac{\ddot{\alpha}_2}{\alpha_2} - \frac{2l}{(l+1)}\frac{\dot{\alpha}_2^2}{\alpha_2} = -\frac{\alpha_2^{2-4l}}{2}$ (30)
Integrating equation (30) we get:-
$\frac{d\alpha_2}{dt} = [\frac{(l+1)}{4(1-l+l^2)}\alpha_2^{4(1-l)} + l_1\alpha_2^{\frac{-4l}{l+1}}]^{1/2}$ (31)
Here l_1 is the constant of integration using equation (31) and after suitable transformation of co-ordinates metric (1) can be written as
$ds^2 = -[\frac{(l+1)}{4(1-l+l^2)}\tau^{4(1-l)} + l_1\tau^{\frac{-4l}{l+1}}]^{-1}d\tau^2 +$
$\tau^2(dX_1 + X_3 dX_2)^2 +$
$\tau^{2l}(dX_2^2 + dX_3^2)$ (32)
The pressure p, the energy density ρ, the string tension density μ, the expansion θ, the shear scalar σ and the particle density (ρ_p) for (32) are given by
$\rho = -p = \frac{l^2(l+3)}{2(1-l-l^2)}\tau^{2-4l} + l(l+2)l_1\tau^{\frac{-2(1+3l)}{(l+1)}}$
(33)
$\mu = \frac{(l+1)(1+l-2l^2-l^3)}{2(1-l-l^2)}\tau^{2-4l} + \frac{2l(l+2)(1-l)}{(1+l)}l_1\tau^{\frac{-2(1+3l)}{(l+1)}}$
(34)
$\theta = (2l+1)[\frac{l+1}{4(1-l-l^2)}\tau^{2-4l} + l_1\tau^{\frac{-2(1+3l)}{(l+1)}}]^{\frac{1}{2}}$
(35)
$\sigma^2 = \frac{(l-1)^2}{3}[\frac{(l+1)}{4(1-l-l^2)}\tau^{2-4l} + l_1\tau^{\frac{-2(1+3l)}{(l+1)}}]$
(36)
$\rho_p = [\frac{(3l^3-9l-1)}{4(1-l-l^2)}\tau^{2-4l} + l_1\tau^{\frac{-2(1+3l)}{(l+1)}}]$
(37)

5. DISCUSSION FOR CASE (1)

If $l > \frac{1}{2}$ and k_1 as negative integer value. We find that the energy conditions $\rho \geq 0$, $\rho_p \geq 0$; are satisfied. The model starts with a big bang at $\tau = 0$.If time is increases expansion in the model decreases.The expansion steps at, $\tau = \infty$ when $\tau \to 0$, then $\rho \to \infty$, $\mu \to \infty$. When $\tau \to \infty, \rho \to 0, \mu \to 0$. Also $p \to \infty$,When $\tau \to 0$ and $p \to 0$ When $\tau \to \infty$. The above calculation shows that when $\tau \to 0$ then $V \to 0$ and when $\tau \to \infty$ then $V \to \infty$. These results represent the universe starts expanding with zero volume and blow up at infinite past and future.

6. DISCUSSION FOR CASE (2)

If $l = \frac{1}{2}$ *and* $l_1 =$ positive constant we find that the energy condition $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. If time is increases then expansion in model decreases. If $\tau = \infty$, then expansion scalar is zero i.e. Expansion stops at $\tau = \infty$. When $\tau \to 0$ then $\rho \to \infty$ $\mu \to \infty$. When $\tau \to \infty$, $\rho \to 0, \mu \to 0$. Also $p \to \infty$, When $\tau \to 0$ *and* $p \to 0$ When $\tau \to \infty$
Using equation (36) and (35) we get
$$\frac{\sigma^2}{\theta^2} = \frac{(l-1)^2}{3(2l+1)} = K'(Constant)$$
The Model does not approach isotropy in general.

REFERENCES

[1] P. S. Letelier, phys. Rev 1983, D28, pp. 2414-2419, 1983..
[2] R. Bali, and Anjali, astrophys. space Sci.,302, 201, 206, 2006.
[3] B. YaZel'sdovich, I. Yu Kokzarev, and L.B. okun, zn. Sov. Phys. JETP, 40 1,5 ,1975.
[4] R. Bali and S. Dave, Astrophys. space Sci.,2 88,503, 509, 2003.
[5] J. Stachel, Phys. Rev., D21, pp. 2171-2181, 1980.
[6] E. Asseo and H sol. Phys. Rep.,148, 307, 1987.
[7] R. Bali and R. D. Upadhayay. Astrophys. and Space- Science., 287, 288, 292, 2003.
[8] A. Pradhan and S. S. Kumhar IJTP. 48(5), pp. 1466-1477, 2009.
[9] A. Pradhan, H. Amirhashchi and M. K. Yadav, Fizika. B18, pp. 35-42, 2009.
[10] V. U. M Rao, M. V. Shanti and T Vinutha Astroy and Space sc. 3014, pp. 73-77, 2008..
[11] A. Tyagi, P. Jain and K. Sharma, JRAPS. 8, pp. 173-178, 2009.